Libyan International Medical Universaty
Faculty of Buisness Adminstration

Operation Research

PERT & CPM

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Overview

Operations Research, also called Decision Science or Operations Analysis, is the study of applying mathematics to business questions. As a sub-field of Applied Mathematics.

Network Analysis (PERT) & (CPM)

01

In Network Analysis, an application is represented as a graph to be optimized. In Graph theory in general, graphs exist of nodes and edges. Each node is a point and each edge indicates whether there is a connection between certain different nodes.

Two methods of network analysis that can help here are the Critical Path Method (CPM) and the Program Evaluation & Review Technique (PERT) method.

Methods of Network Analysis - Definition

(PERT)

is a project management technique, used to manage uncertain activities of a project.

(CPM)

is a statistical technique of project management that manages well defined activities of a project.

Basic scheduling computations

The notations used are:

- **Ei = Earliest occurrence time of event i**
- **Lj = Latest allowable occurrence time of event**
- **Dij = Estimated completion time of activity**
- **Es = Earliest starting time of activity**
- **Ef = Earliest finishing time of activity**
- **Ls = Latest starting time of activity**
- **Lf = Latest finishing time of activity**

(i, j) = Activity with tail event (i), and head event (j)

Determination of earliest time : Forward pass computation

1 . Step - The computation begins from the start node and move towards the end node. For easiness, the forward pass computation starts by assuming the earliest occurrence time of zero for the initial project event

2 .Step - Earliest starting time of activity (i, j) is the earliest event time of the tail end event

(Es)ij = Ei i. Earliest finish time of activity (i, j) is the earliest starting time + the activity time i.e. (Ef)ij = (Es)ij + Dij or (Ef)ij = Ei + Dij iii. Earliest event time for event j is the maximum of the earliest finish times of all activities ending in to that event i.e. Ej = max [(Ef)ij for all immediate predecessor of (i, j)] or Ej = max [Ei + Dij]

Backward pass computation (for latest allowable time)

1 .Step - For ending event assume E = L. Remember that all E's have been computed by forward pass computations.

2 .Step - Latest finish time for activity (i, j) is equal to the latest event time of event j i.e. $(L_f)_{ij} = L_j$

3 .Step - Latest starting time of activity (i, j) = the latest completion time of (i, j) – the activity time or $(L_s)_{ij} = (L_f)_{ij} - D_{ij}$ or $(L_s)_{ij} = L_j - D_{ij}$

4 .Step - Latest event time for event 'i' is the minimum of the latest start time of all activities originating from that event i.e. $L_i = \min [(L_s)_{ij}$ for all immediate successor of (i, j)$] = \min [(L_f)_{ij} - D_{ij}] = \min [L_j - D_{ij}]$

Determination of total float

- **Total Float**

The amount of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time.

Mathematically:

(Tf)ij = (Latest start – Earliest start) for each activity or
(Latest finish – Latest finish)

Determination of critical path

(i) Critical activity - The activities with zero total float are known as critical activities. In other words an activity is said to be critical if a delay in its start will cause a further delay in the completion date of the entire project.

(ii) Critical path - The sequence of critical activities in a network is called critical path. The critical path is the longest path in the network from the starting event to ending event and defines the minimum time required to complete the project.

Dummy activity

A Dummy activity is a type of operation in a project network which neither requires any time nor any resource. It is an imaginary activity shown in a project network to identify the dependence among operation.

Critical Path Method

02

Critical Path Method

- **CPM estimates the maximum and minimum time required to complete a project.**

- **With the CPM, you will need to estimate all task completion times and identify all dependable variables that could influence your project’s progress.**

- **With the CPM, you will include milestones that need to be met at certain times to ensure your project is on track.**

Example that shows how to calculate critical path

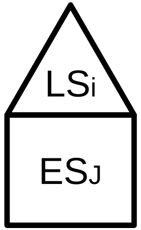
Consider the eight activities associated with the fabrication of a steel component shown in Table .

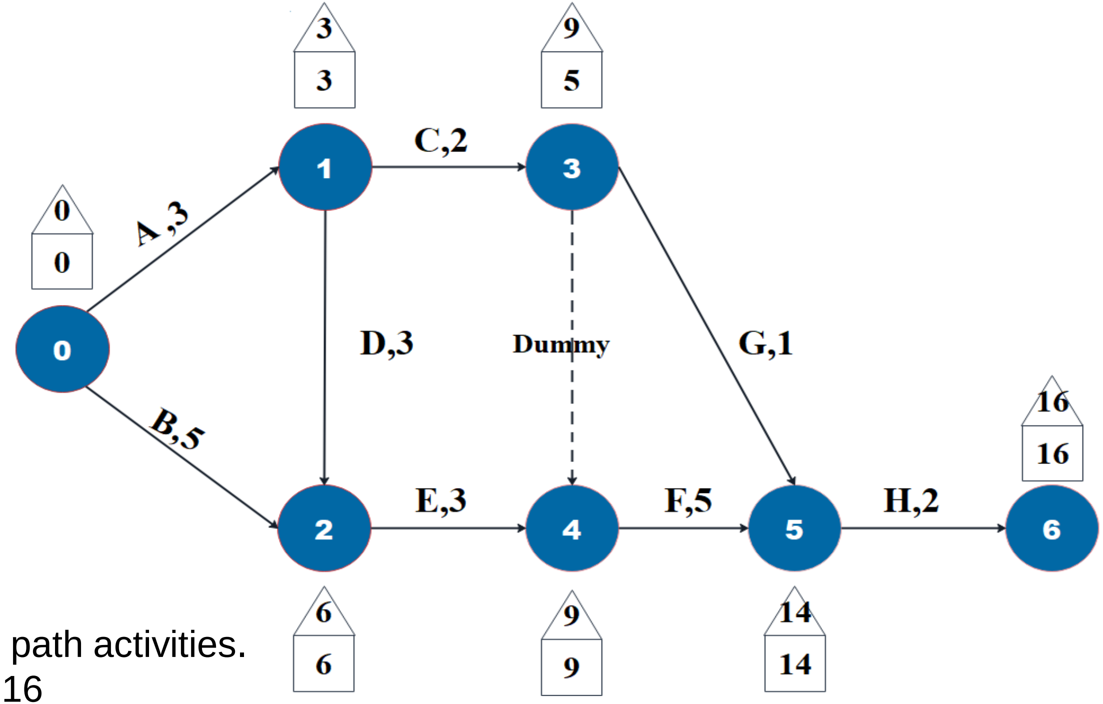
Activity	Duration	Predecessors
A	3	-
B	5	-
C	2	A
D	3	A
E	3	B,D
F	5	C,E
G	1	C
H	2	F,G

Figure below shows the network diagram associated with these eight activities.

$$Esj = Esi + D \quad , \quad Esj = Max\ (Esi + D) \quad \{FORWARD\}$$

$$Lci = Lcj - D \quad , \quad Lci = Min\ (Lcj - D) \quad \{BACKWARD\}$$





A-D-E-F-H. are the critical path activities.
Project completion time = 16

03

Program Evaluation & Review Technique

PERT

PERT is a way to schedule the flow of tasks in a project and estimate the total time taken to complete it. This technique helps represent how each task is dependent on the other.

To schedule a project using PERT, one has to define activities, arrange them in an orderly manner and define milestones. You can calculate timelines for a project on the basis of the level of confidence.

Probability Times

Which is the shortest assuming period for the completion of the activity.

Optimistic timing

This is the natural period for completing the activity.

Most-likely timing

Which is the longest assuming period for the completion of the activity.

Pessimistic timing

Equation

Expected Time = optimistic time + 4(most likely time) + pessimistic time / 6

T = a + 4m + b / 6 Standard Deviation

(σ) = b – a / 6

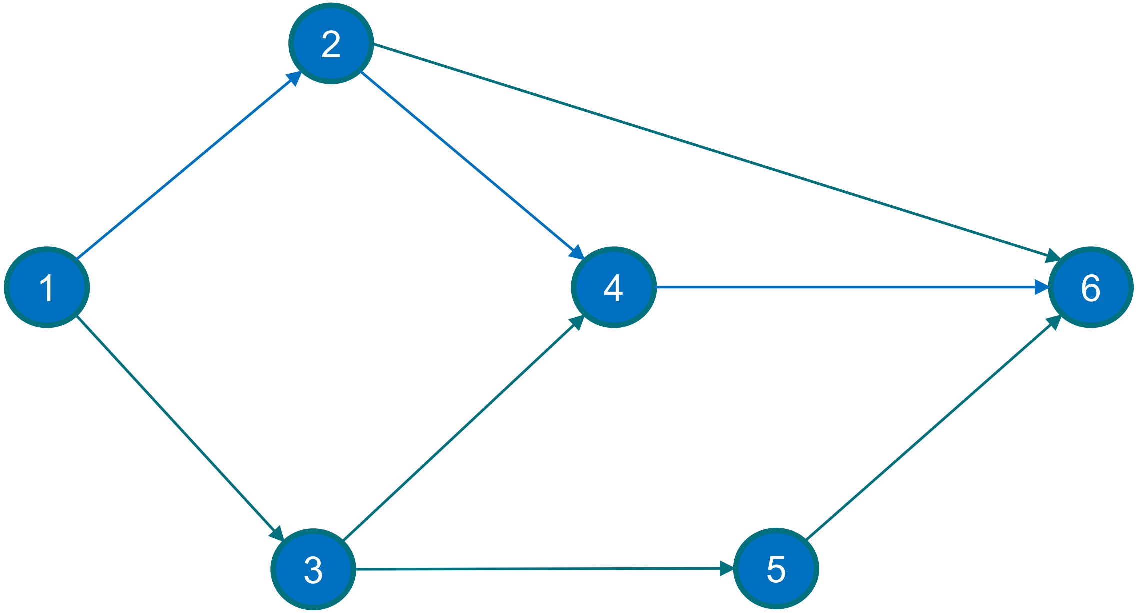
For the given activities determine:

1. Critical path using PERT.
2. Calculate the standard deviation for each activity.
3. Calculate the probability of completing the project in 26 days.

Activity	t_o	t_m	t_p
1-2	6	9	12
1-3	3	4	11
2-4	2	5	14
3-4	4	6	8
3-5	1	1.5	5
2-6	5	6	7
4-6	7	8	15
5-6	1	2	3

solution

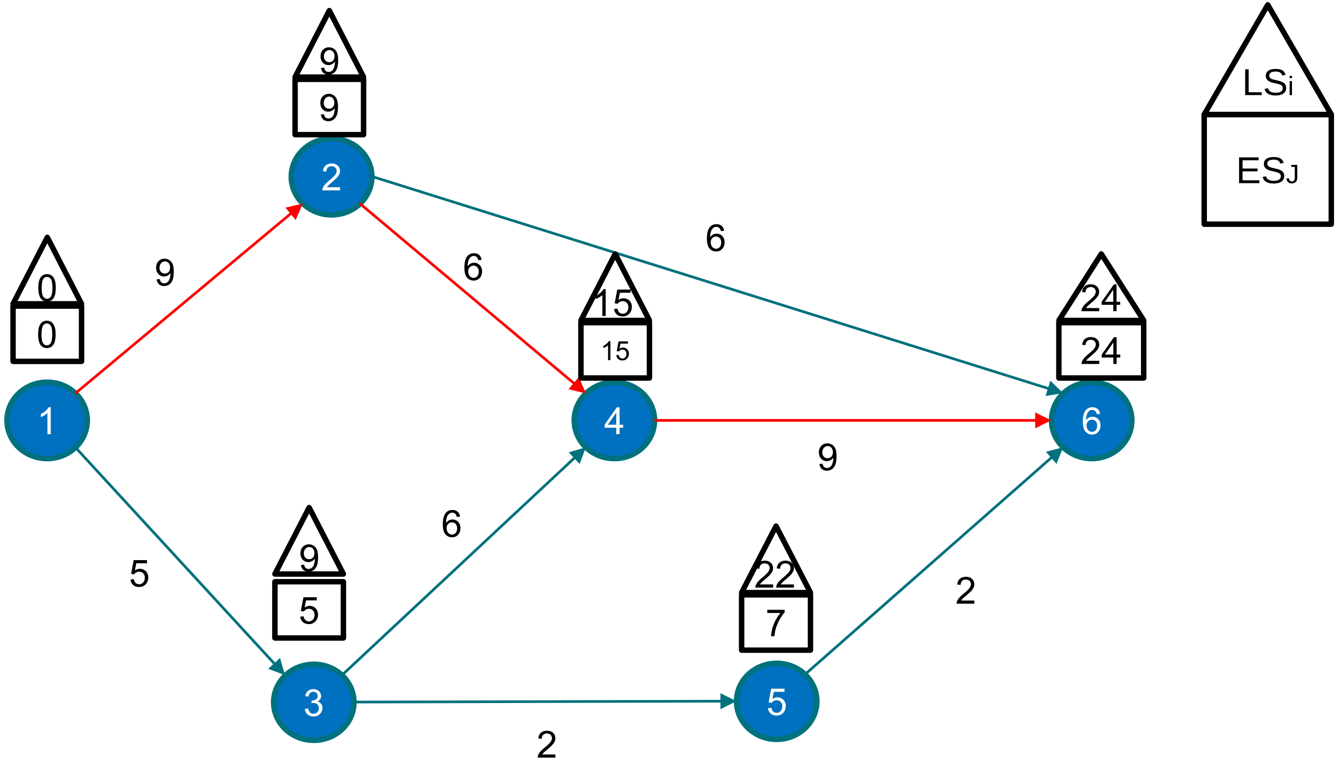
First of all draw the network diagram for given data as shown below:



For each given activity we will calculate the expected time as follows:

Activity	t_o	t_m	t_p	te= $\frac{t_o + 4t_m + t_p}{6}$	σ
1-2	6	9	12	$\frac{6 + 4 * 9 + 12}{6} = 9$	$\frac{12-6}{6}$=1
1-3	3	4	11	5	1.33
2-4	2	5	14	6	2
3-4	4	6	8	6	0.67
3-5	1	1.5	5	2	0.67
2-6	5	6	7	6	0.33
4-6	7	8	15	9	1.33
5-6	1	2	3	2	0.33

Here the critical path is along the activities 1-2, 2-4, 4-6 So the critical path is. 1-2-4-6 Following diagram is prepared to show critical path along with EST and LFT .



The critical path = 1-2-4-6 with time duration (*tcp*) of 24 days.

Here standard deviation is calculated for activities of critical path. So we get

$$\sigma = \sqrt{V1 + V2 + V3}$$
$$\sigma = \sqrt{1 + 4 + 1.778}$$
$$=2.6034$$

Now the probability of completion of project in that given time (t) of 26 days, can be calculate by below formula

$$Z= \frac{t-tcp}{\sigma} = \frac{26-24}{2.6034} = 0.77$$

Using table in Appendix-B, we get probability =77.94%

As you can see below in Appendix-B the first column Z that is the probability we find out in the example through formula. The second column ψ (z) represent the probability in percentage (%).

Appendix - B / ”Z” Table

APPENDIX - B

Proportion of total area under the normal curve from $-\infty$ to z,

where $z = \dfrac{x-\mu}{\sigma}$.

z	ψ (z)	z	ψ (z)	z	ψ (z)
0.00	0.5000	0.43	0.6664	0.85	0.8023
0.01	0.5040	0.44	0.6700	0.86	0.8051
0.02	0.5080	0.45	0.6736	0.87	0.8078
0.03	0.5120	0.46	0.6772	0.88	0.8106
0.04	0.5160	0.47	0.6808	0.89	0.8133
0.05	0.5199	0.48	0.6844	0.90	0.8159
0.06	0.5239	0.49	0.6879	0.91	0.8186
0.07	0.5279	0.50	0.6915	0.92	0.8212
0.08	0.5319	0.51	0.6950	0.93	0.8238
0.09	0.5359	0.52	0.6985	0.94	0.8264
0.10	0.5398	0.53	0.7019	0.95	0.8289
0.11	0.5438	0.54	0.7054	0.96	0.8315
0.12	0.5478	0.55	0.7088	0.97	0.8340
0.13	0.5517	0.56	0.7123	0.98	0.8365
0.14	0.5557	0.57	0.7157	0.99	0.8389
0.15	0.5596	0.58	0.7190	1.00	0.8413
0.16	0.5636	0.59	0.7224	1.01	0.8438
0.17	0.5675	0.60	0.7257	1.02	0.8461
0.18	0.5714	0.61	0.7291	1.03	0.8485
0.19	0.5753	0.62	0.7324	1.04	0.8508
0.20	0.5793	0.63	0.7357	1.05	0.8531
0.21	0.5832	0.64	0.7389	1.06	0.8554
0.22	0.5871	0.65	0.7422	1.07	0.8577
0.23	0.5910	0.66	0.7454	1.08	0.8599
0.24	0.5948	0.67	0.7486	1.09	0.8621
0.25	0.5987	0.68	0.7517	1.10	0.8643
0.26	0.6026	0.69	0.7549	1.11	0.8665
0.27	0.6064	0.70	0.7580	1.12	0.8686
0.28	0.6103	0.71	0.7611	1.13	0.8708
0.29	0.6141	0.72	0.7642	1.14	0.8729
0.30	0.6179	0.73	0.7673	1.15	0.8749
0.31	0.6217	0.74	0.7703	1.16	0.8770
0.32	0.6255	0.75	0.7734	1.17	0.8790
0.33	0.6293	0.76	0.7764	1.18	0.8810
0.34	0.6331	0.77	0.7794	1.19	0.8830
0.35	0.6368	0.78	0.7823	1.20	0.8849
0.36	0.6406	0.79	0.7852	1.21	0.8869
0.37	0.6443	0.80	0.7881	1.22	0.8888
0.38	0.6480	0.81	0.7910	1.23	0.8907
0.39	0.6517	0.82	0.7939	1.24	0.8925
0.40	0.6554	0.83	[illegible]	1.25	0.8944
0.41	0.6591	0.84	0.7995	1.26	0.8962
0.42	0.6628			1.27	0.8980
				1.28	0.8997
				1.29	0.9015

Crashing Time 04

Project completion times may need to be shortened because:

- Different deadlines
- Promised completion dates

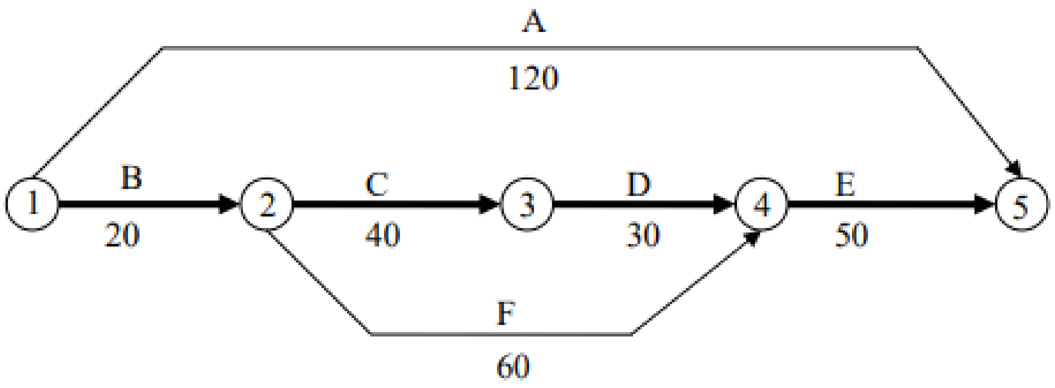
Reduced project completion time is "crashing"

Crashing Example:

The network and durations given shows the normal schedule for a project. You can decrease (crash) the durations at an additional expense.

The Table given summarizes the time-cost information for the activities.

The owner wants you to finish the project in 110 days. Find the minimum possible cost for the project if you want to finish it on 110 days.



Activity	Normal duration (days)	Crash Duration (days)	Normal Cost	Crash Cost
A	120	100	12000	14000
B	20	15	1800	2800
C	40	30	16000	22000
D	30	20	1400	2000
E	50	40	3600	4800
F	60	45	13500	18000

Solution:

Assume that the duration-cost relationship for each activity is a single linear, continuous function between the crash duration and normal duration points. Using the normal duration (ND), crash duration (CD), normal cost (NC), and crash cost (CC), the crash cost slope for each activity can be determined as follows;

$$S_A = \frac{CC - NC}{ND - CD}$$

$$S_A = \frac{14000 - 12000}{120 - 100} = \$100/day$$

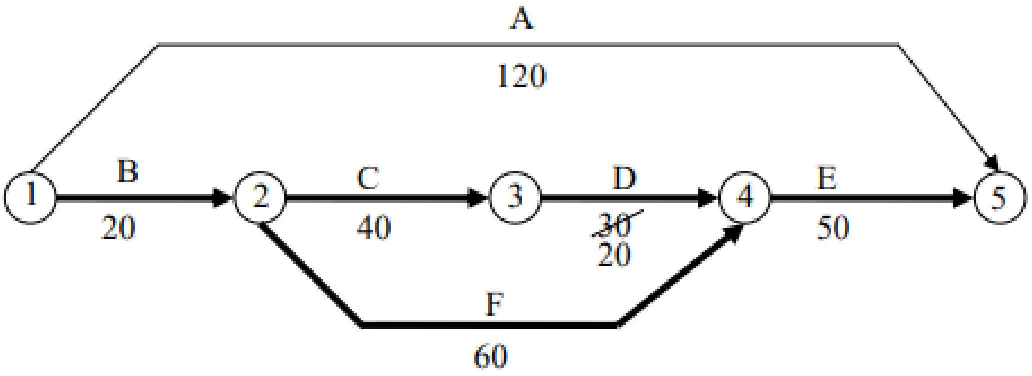
SB = $200/day
SC = $600/day
SD = $60/day
SE = $120/day
SF = $300/day

Cont.

The normal cost for the project is the sum of a normal cost for each activity. The normal cost for the project is $48300 and the normal duration is 140 days.

The activity which should be crashed is the one on the critical path which will add the least amount to the overall project cost. This will be the activity with the flattest or least-cost slope. The duration can be reduced as long as the critical path is not changed or a new critical path is created. In addition, the activity duration cannot be less than the crash duration.

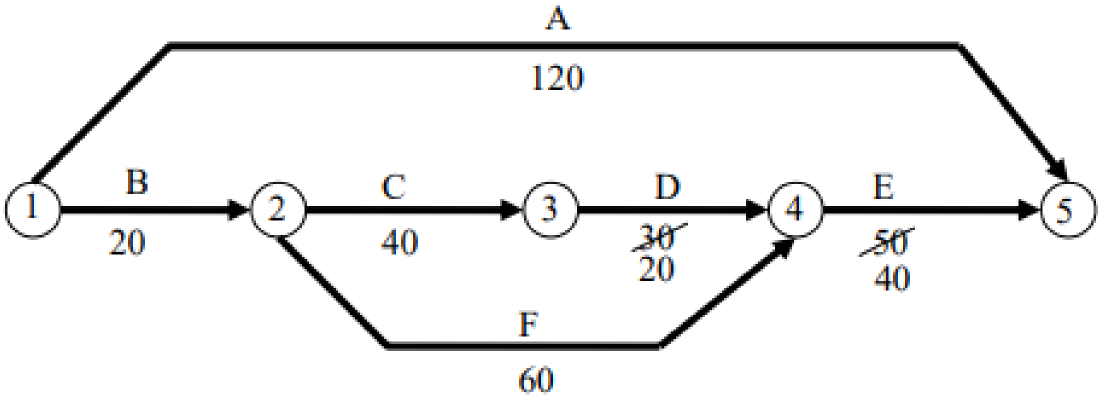
SD = $60/day (least-cost slope) Maximum of 10 days can be cut from this schedule by reducing the duration of activity D to the crash duration of 20 days.



Overall duration is 130 days and there are multiple critical paths (B-F-E and B-C-D-E). Total project cost at this duration is the normal cost of $48300 plus the cost of crashing the activity D by 10 days (60 * 10 = $600) for a total of $48900.

Cont.

The next activity to be crashed would be the activity E, since it has the least-cost slope ($120 per day) of any of the activities on the critical path. Activity E can be crashed by a total of 10 days. Crashing the activity E by 10 days will cost an additional $120 per day or $1200.

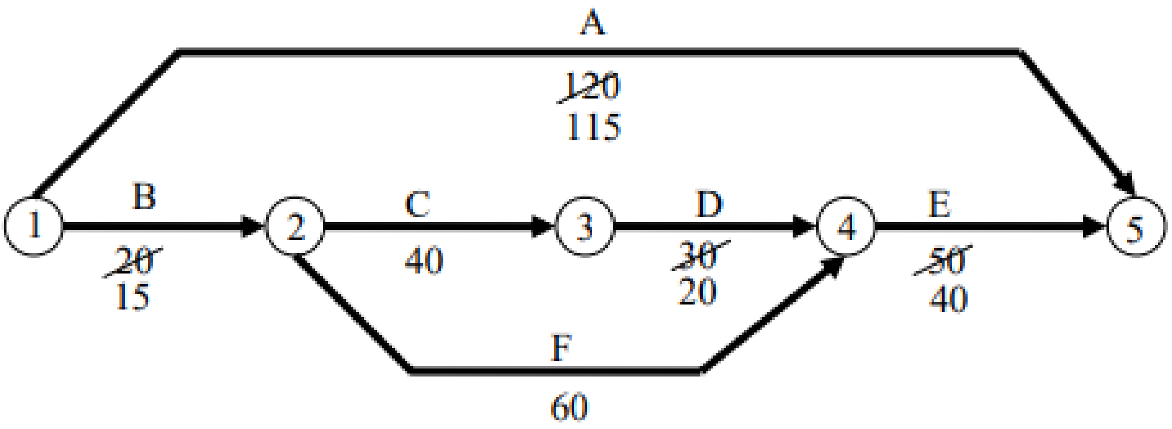


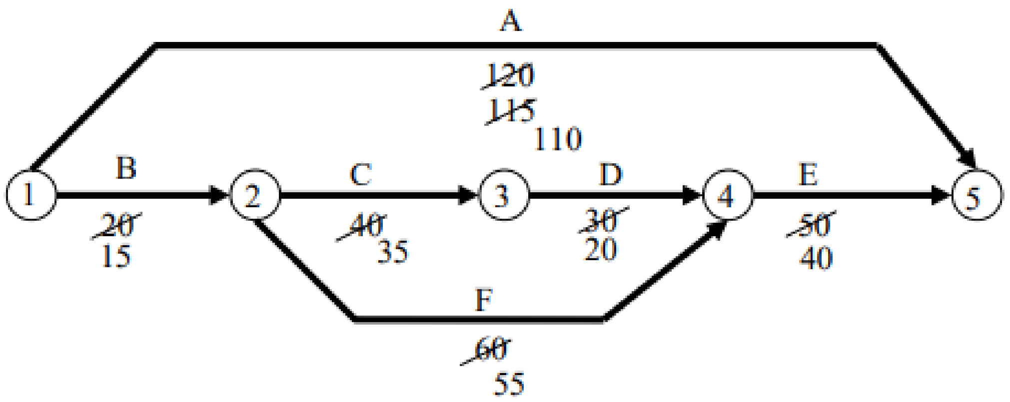
Cont.

The project duration is now 120 days and the total project cost is $50100. There are now three critical paths (A, B-C-D-E, and B-F-E). The next stage of crashing requires a more through analysis since it is impossible to crash one activity alone and achieve a reduction in the overall project duration. Activity A is paired with each of the other activities to determine which has the least overall cost slope for those activities which have remaining days to be crashed.

Activity A ($100) + activity B ($200)
Activity A ($100) + activity C ($600) + activity F ($300)

The least-cost slope will be activity A + activity B for a cost increase of $300 per day. Reducing the project duration by 5 days will add 5*300 = $1500 dollar crashing cost and the total project cost would be $51600. Activity B can not be crashed any more.





Final step in crashing the project to 110 days would be accomplished by reducing the duration of activity A by 5 days to 110 days, reducing activity C by 5 days to 35 days, and reducing activity F by 5 days to 55 days. The combined cost slope for the simultaneous reduction of activity A, activity C, and activity F would be $1000 per day. For 5 days of reduction this would be an additional $5000 in total project cost.

The total project cost for the crashed schedule to 110 days of duration would be $56600.

Differences between CPM & PERT

05

BASIS FOR COMPARISON	PERT	CPM
Meaning	PERT is a project management technique, used to manage uncertain activities of a project.	CPM is a statistical technique of project management that manages well defined activities of a project.
What is it?	A technique of planning and control of time.	A method to control cost and time.
Orientation	Event-oriented	Activity-oriented
Evolution	Evolved as Research & Development project	Evolved as Construction project
Model	Probabilistic Model	Deterministic Model
Focuses on	Time	Time-cost trade-off
Estimates	Three time estimates	One time estimate

Network Example

06

step-by-step

In the question here, we have three objectives:

1) Draw the network diagram.

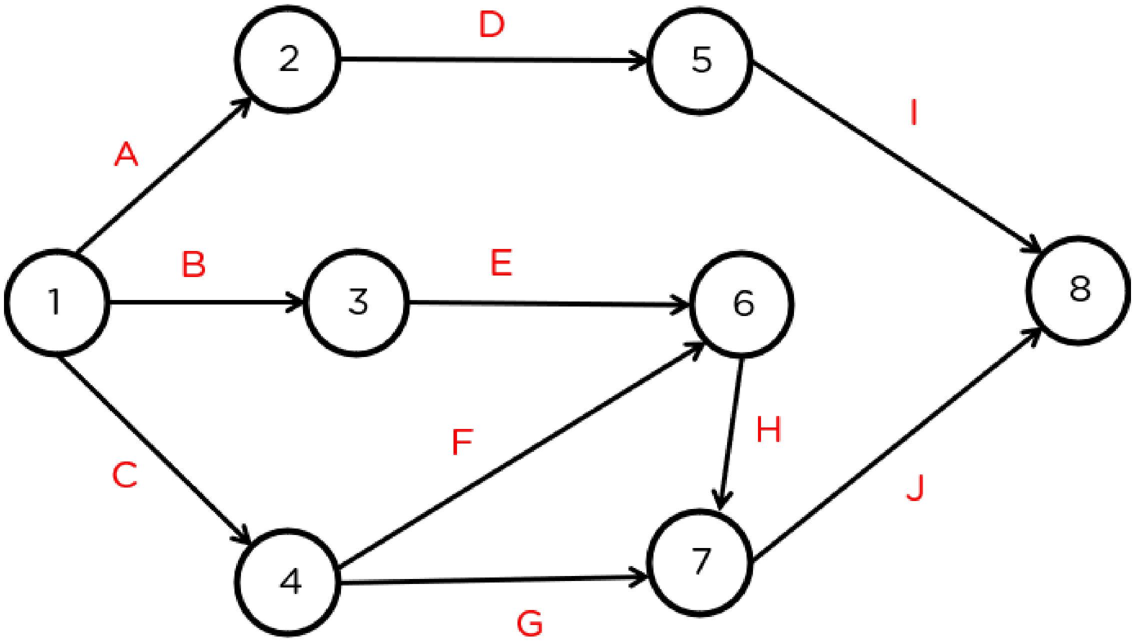
2) Find the mean and variance.

3) Find the critical path and estimated time of completion.

The Given Activities

Activities	Immediate Predecessor	Optimistic Time	Most Likely Time	Pessimistic Time
A	-	6	7	8
B	-	3	5	7
C	-	4	7	10
D	A	2	3	4
E	B	3	4	11
F	C	4	8	12
G	C	3	3	9
H	E, F	6	6	12
I	D	5	8	11
J	H, G	3	3	9

Now, let's draw the network diagram.



Now that we've created the network diagram, let's move ahead. Next, as part of the PERT analysis, let's have a look at how to determine the mean and variance.
the mean, which is also the estimated time can be determined using the formula:

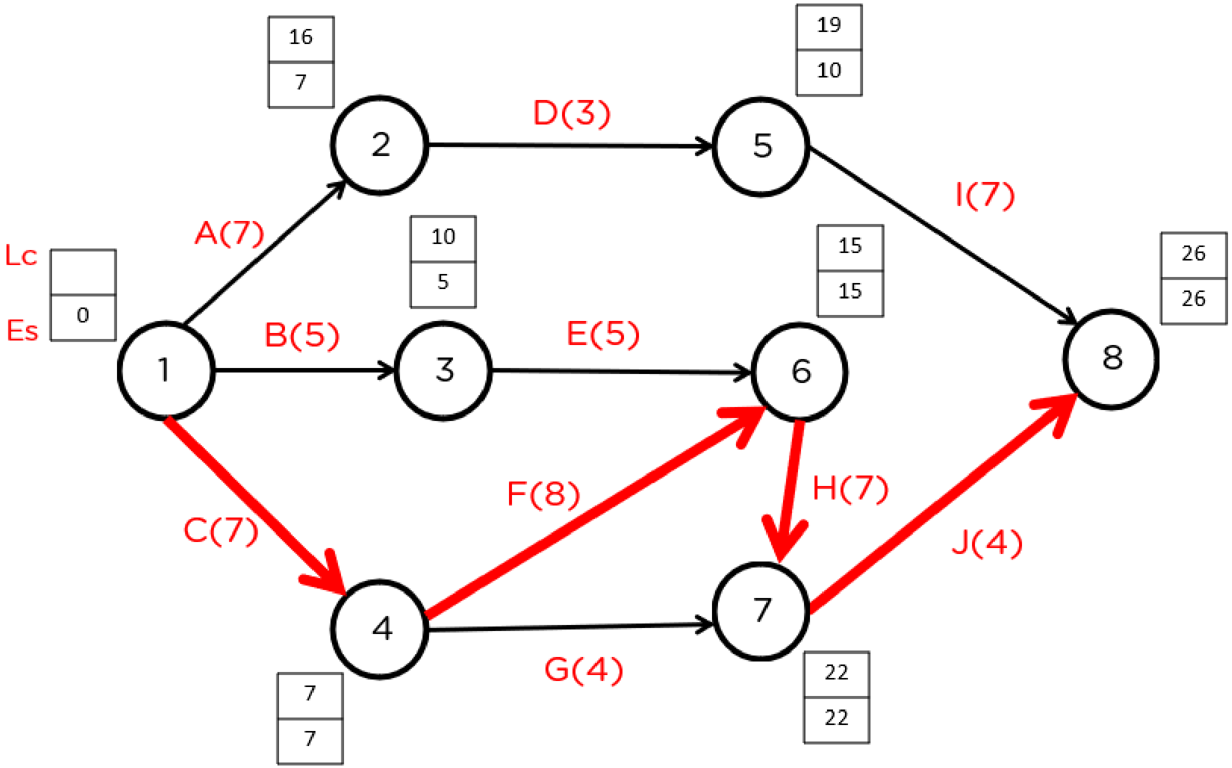
$$T_e = \frac{T_0 + 4Tm + Tp}{6}$$

We can calculate the variance using this formula:

$$\sigma^2 = \left(\frac{T_P - T_0}{6}\right)^2$$

Calculated Table

Activities	Immediate Predecessor	Optimistic Time	Most Likely Time	Pessimistic Time	Mean	Variance
A	-	6	7	8	7	0.11
B	-	3	5	7	5	0.44
C	-	4	7	10	7	1
D	A	2	3	4	3	0.11
E	B	3	4	11	5	1.77
F	C	4	8	12	8	1.77
G	C	3	3	9	4	1
H	E, F	6	6	12	7	1
I	D	5	8	11	7	1
J	H, G	3	3	9	4	1



From the diagram, we can see that nodes that satisfy the requirements are:
1 - 4 - 6 - 7 - 8 or C - F - H – J

The estimated time is: 7 + 8 + 7 + 4 = 26 days.

Conclusion

In conclusion if you as a project manager are working with a project for the first time it would probably be best to use PERT. After using the method and getting more consistent completion times for activities, they at that point can become the duration of those activities using CPM. While both methods were invented separate from each other I observed a definite link between the two.

Network Analysis is useful in many living application tasks. It helps us in deep understanding the structure of a relationship in social networks, a structure or process of change in natural phenomenon, or even the analysis of biological systems of organisms.

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THANK YOU!

Sample text, replace with your own.